

Division Point

Goal:

- to understand the concept of a **division point**
- to determine the coordinates of a division point
- to recognize when a ratio is expressed as **part-to-part** versus when it is expressed as **part-to-whole**

A line segment can be divided into pieces:



In this case point C is called a **division point**.

Point C divides \overline{AB} into two pieces and the lengths of these pieces can be expressed as a ratio.



Here we can see that:

$$m \overline{AC} = 8 \text{ units} \quad \text{and} \quad m \overline{CB} = 2 \text{ units}$$

the ratio in which C divides AB can be expressed in two ways:

1) part-to part:

or

2) part-to-whole

$$\frac{m \overline{AC}}{m \overline{BC}} = \frac{8}{2} = \frac{4}{1}$$

$$4:1$$

$$\frac{m \overline{AC}}{m \overline{AB}} = \frac{8}{10} = \frac{4}{5}$$

$$4:5$$

notice that the order of points matters.

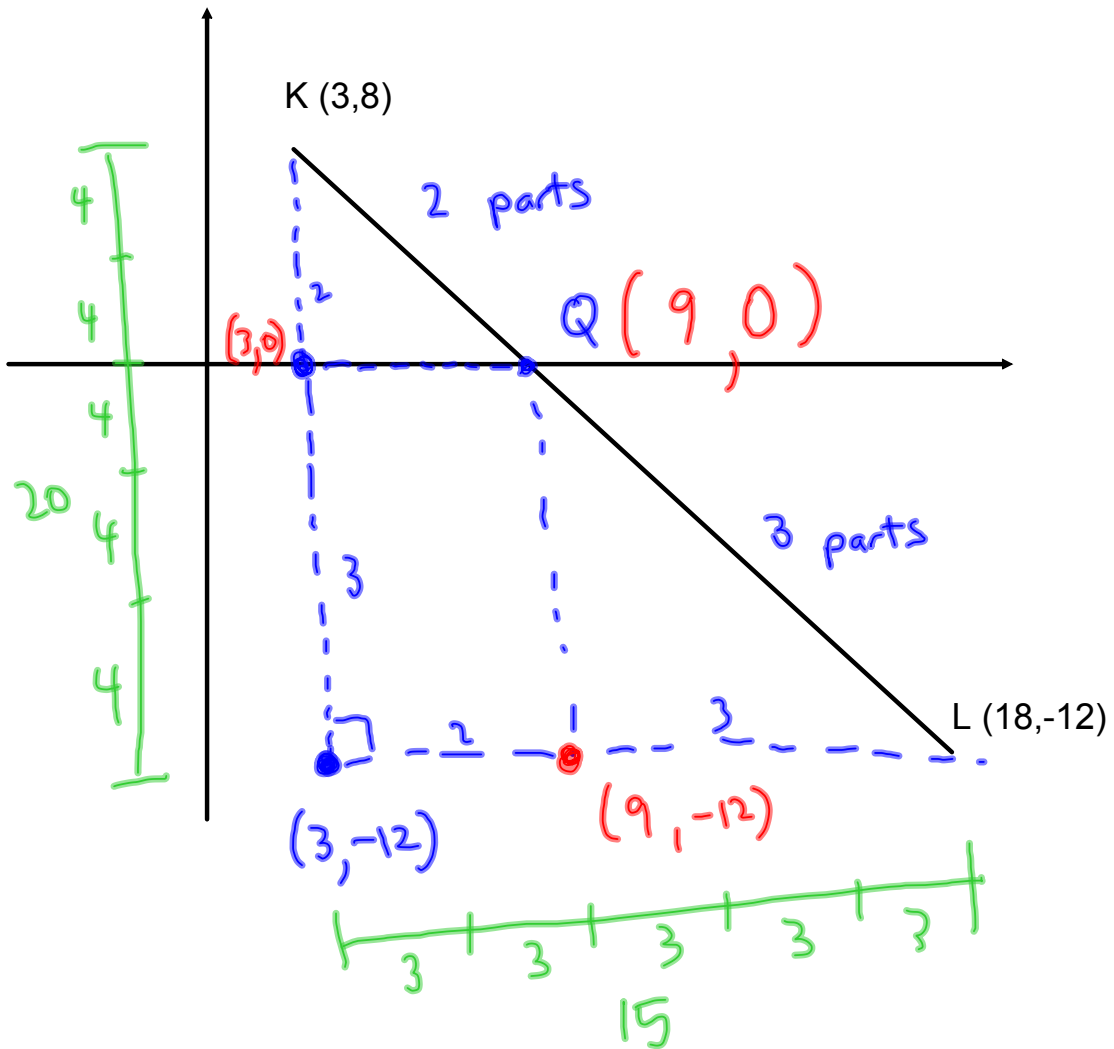
Point C

part-to-part of \overline{BA} 1:4

part-to-whole of \overline{BA} 1:5

If the ratio is known it can be used to find the coordinates of a division point. *part-to-part*

Ex: Point Q divides line segment \overline{KL} into a ratio of 2:3. Find the coordinates of point Q given $K(3,8)$ and $L(18,-12)$.



The formula for finding the coordinates of a division point $P(x_p, y_p)$ is:

$$X_p = X_1 + \frac{a}{a+b} (X_2 - X_1)$$

X_1 : is the x-coord. of start point

X_2 : is the x-coord. of end point

$\frac{a}{a+b}$, is the part-to-whole ratio

$$Y_p = Y_1 + \frac{a}{a+b} (Y_2 - Y_1)$$

Notice the formula requires that the ratio be expressed part-to-whole.

part-to-part $a:b$

Ex: Line segment EF has endpoints E(-33,11) and F(-1,-9).
Find the coordinates of the division point Z that:

a) is located $\frac{3}{4}$ of the way along \overline{EF} . *part-to-whole*

$$x_2 = x_1 + \frac{a}{a+b}(x_2 - x_1) = -33 + \frac{3}{4}(-1 - (-33))$$

$$= -33 + \frac{3}{4}(32) = -33 + 24 = -9$$

$$y_2 = y_1 + \frac{a}{a+b}(y_2 - y_1) = 11 + \frac{3}{4}(-9 - 11)$$

$$= 11 + \frac{3}{4}(-20) = 11 - 15 = -4$$

$$Z(-9, -4)$$



b) that divides \overline{FE} into a ratio of 1:3.

part-to-part 1:3

part-to-whole: $\frac{1}{1+3} = \frac{1}{4}$

$$x_2 = x_1 + \frac{a}{a+b}(x_2 - x_1)$$

$$= -1 + \frac{1}{4}(-33 - (-1))$$

$$= -1 + \frac{1}{4}(-32)$$

$$= -1 - 8 = -9$$

$$y_2 = y_1 + \frac{a}{a+b}(y_2 - y_1)$$

$$= -9 + \frac{1}{4}(11 - (-9))$$

$$= -9 + \frac{1}{4}(20)$$

$$= -9 + 5$$

$$= -4$$

$$Z(-9, -4)$$

Homework p.18 #10, 13

14. $A(15, 6)$ distance = 17

$B(0, y)$ is located on y -axis

$$17 = \sqrt{(0-15)^2 + (y-6)^2} \quad \text{OR} \quad c^2 = a^2 + b^2$$

$$17 = \sqrt{(-15)^2 + (y-6)^2}$$

$$17 = \sqrt{225 + (y-6)^2}$$

$$289 = 225 + (y-6)^2$$

$$289 - 225 = (y-6)^2$$

$$64 = (y-6)^2$$

$$\sqrt{64} = y-6$$

$$8 = y-6$$

$$14 = y$$

$$17^2 = 15^2 + b^2$$

$$289 = 225 + b^2$$

$$64 = b^2$$

$$8 = b$$

$$b = y-6$$

$$8 = y-6$$

$$y = 14$$