Name: _____

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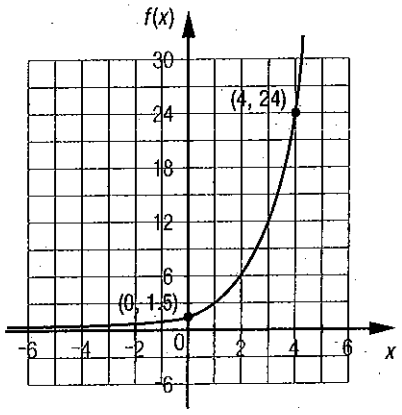
CONSOLIDATION 4.3

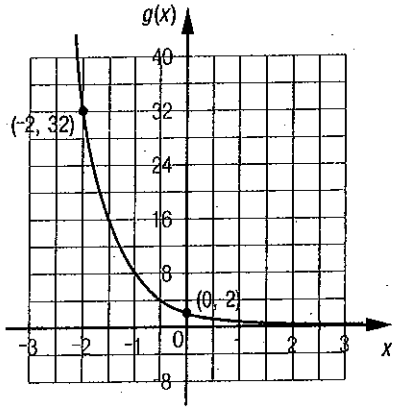
## Exponential functions

**1** Determine the rule for the following exponential functions:

**a)**




**b)**




$f(x) =$ _____ $g(x) =$ _____

**c)**

| $x$ | $h(x)$ |
|---|---|
| 0 | -4 |
| 1 | -8 |
| 2 | -16 |
| 3 | -32 |
| 4 | -64 |

**d)**

| $x$ | $i(x)$ |
|---|---|
| -2 | $\frac{1}{27}$ |
| -1 | $\frac{1}{9}$ |
| 2 | 3 |
| 4 | 27 |
| 5 | 81 |

$h(x) =$ _____ $i(x) =$ _____

**2** The growth of a certain number of bacteria is under laboratory observation. The growth can be modelled by the function $f(x) = 250(2)^x$ where $f(x)$ represents the number of bacteria and $x$ represents the number of hours which have passed since the observation began.

**a)** How many bacteria were there at the beginning of the observation period? _____

**b)** How many bacteria were there after 30 min? _____

**c)** How many bacteria were there after 2 h? _____

**d)** How long will it take before there are 4000 bacteria? _____

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CONSOLIDATION 4.3 *(cont'd)*

**3** Match each of the rules below to its corresponding curve.

**a)** $f(x) = -10\left(\frac{1}{3}\right)^x$ _____

**b)** $g(x) = -20(3)^x$ _____

**c)** $h(x) = 20\left(\frac{1}{3}\right)^x$ _____

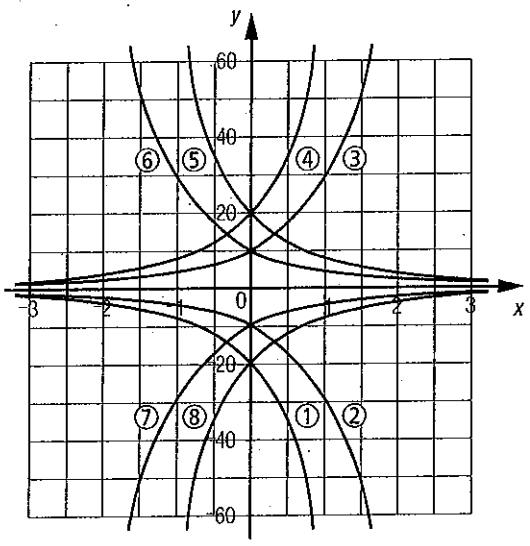
**d)** $i(x) = 10(3)^x$ _____

**e)** $j(x) = -10(3)^x$ _____

**f)** $k(x) = 10\left(\frac{1}{3}\right)^x$ _____

**g)** $l(x) = 20(3)^x$ _____

**h)** $m(x) = -20\left(\frac{1}{3}\right)^x$ _____




**4** One particular property of "superballs" is their ability to bounce by using a large quantity of the energy they have absorbed. Fred bounces a ball several times. After each bounce, the maximum height the ball reaches decreases by 20% in relation to the ball's last bounce.

**a)** If Fred drops the ball from a height of 3 m, find the rule which allows you to calculate the maximum height of the ball as a function of the number of bounces.

_____

**b)** Determine the maximum height of the ball after 4 bounces if Fred drops the ball from a height of 1.5 m.

_____

**c)** From what height has Fred dropped the ball if after 3 bounces the maximum height that the ball reaches is 61.4 cm?

_____

**d)** According to the model associated to this situation, after how many bounces will the ball stop bouncing? Explain your answer.

_____

_____

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**5** A financial institution offers a credit card with an annual interest rate of 18%. In reality, cardholders pay an interest rate of 1.5% compounded monthly.

**a)** What is the real rate of interest paid after one year on a $1,000 purchase?

_____

**b)** This financial institution offers two new credit cards.

Card **1**: 24% interest rate, compounded yearly

Card **2**: 1.9% interest rate, compounded monthly

Which card is more advantageous to the cardholder? Explain your answer.

_____

_____

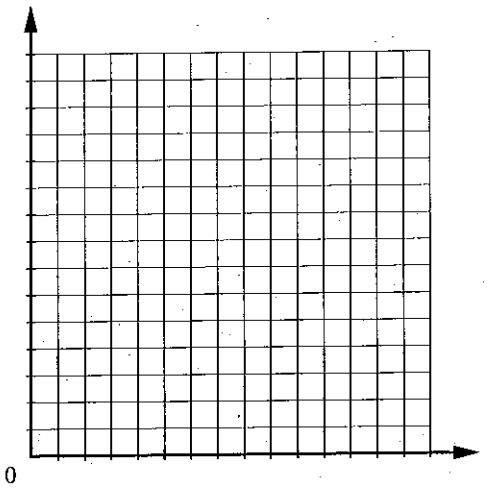
_____

_____

**6** The ordered pairs below represent the value of an investment in relation to the number of years since the investment was made.

(0, 5000), (3, 7600), (5, 11 900),
(7, 13 300), (9, 15 000), (10, 20 200),
(11, 23 200), (13, 32 800), (15, 44 900)

**a)** After you have represented this situation with a scatter plot, draw the curve which best represents the data shown in the graph.




**b)** Determine the rule that allows you to calculate the value of the investment as a function of the number of years since the investment was made.

_____

**c)** What is the function's base, and to what does it correspond to in this situation?

_____

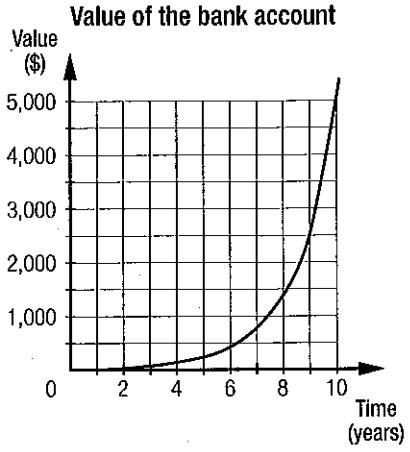
**d)** If the value of the investment continues to grow in the same way, what will it be worth after 18 years?

_____

**b)** 1) Curves for which the base is between 0 and 1 come closer to the *x*-axis as the value of the independent variable increases.

2) Curves for which the value of parameter **a** is negative are reflected about the *x*-axis.

**5. a)**




**b)** $f(x) = 5(2)^x$

**c)** $163,840

## Consolidation 4.3 — Page 19

**1. a)** $f(x) = 1.5(2)^x$ **b)** $g(x) = 2(0.25)^x$

**c)** $h(x) = -4(2)^x$ **d)** $i(x) = \frac{1}{3}(3)^x$

**2. a)** 250 bacteria. **b)** About 353 bacteria.

**c)** 1000 bacteria. **d)** After 4 h.

## Consolidation 4.3 (*cont'd*) — Page 20

**3. a)** ⑦ **b)** ① **c)** ⑤ **d)** ③

**e)** ② **f)** ⑥ **g)** ④ **h)** ⑧

**4. a)** $f(x) = 3(0.8)^x$

**b)** 61.44 cm

**c)** $\approx$ 1.20 m

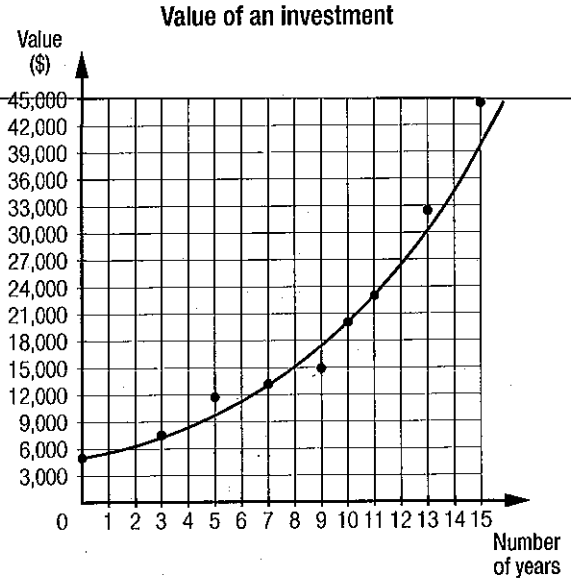
**d)** According to the model, the ball will never stop bouncing because this situation is typical of an exponential function having an asymptote at $y = 0$. In reality, the ball will eventually stop bouncing.

## Consolidation 4.3 (*cont'd*) — Page 21

**5. a)** An approximate interest rate of 19.56%.

**b)** Card **1** is the most advantageous because the annual interest rate on Card **2** is approximately 25.34%.

**6. a)**




**b)** $f(x) = 5000(1.15)^x$

**c)** The base of this function is 1.15. It represents the yearly increase of the investment's value.

**d)** $\approx$ $61,877.27

## Enrichment 4.3 — Page 22

**1.** The equation for the exponential model $y \approx 0.01259(0.999940068)^x$.
Constructing a table of values gives an answer of approximately 157 000 years.

**2.** Set-up two points with coordinates $(m, n)$ and $(c, d)$ such that $\frac{d}{n} < 0$.

If the points belong to an exponential function of the form $y = a(\text{base})^x$, it is possible to write two equations:

$n = a(\text{base})^m$ and $d = a(\text{base})^c$.

The proportion is $\frac{d}{n} = \frac{a(\text{base})^c}{a(\text{base})^m}$ and the equation $(\text{base})^{c-m} = \frac{d}{n} \Rightarrow (\text{base})^{c-m} < 0$.

This equation has no real solution as there is no positive real number raised to either a positive or negative exponent that yields a negative as a solution.

## Activity 1 — Page 23

**a.** 1) With a first-degree polynomial function.

2) With a second-degree polynomial function.

3) With a first-degree polynomial function.

4) With a zero-degree polynomial function.

**b.** With a piecewise function.