

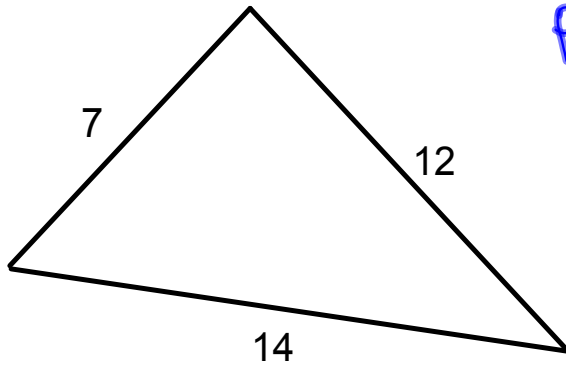
Area of a Triangle

Goal:

- to understand Hero's (Heron's) Formula and the Trigonometric Formula
- to find the area of any triangle

Homework: p. 110 # 6, 9

Hero's Formula is used for the calculation of the area of triangle based on the perimeter of the triangle.



$$P = 7 + 12 + 14 \\ = 33 \text{ units}$$

$$\text{semi-perimeter} = p = \frac{1}{2} P = \frac{1}{2} (33) \\ = 16.5 \text{ units}$$

Hero's formula:

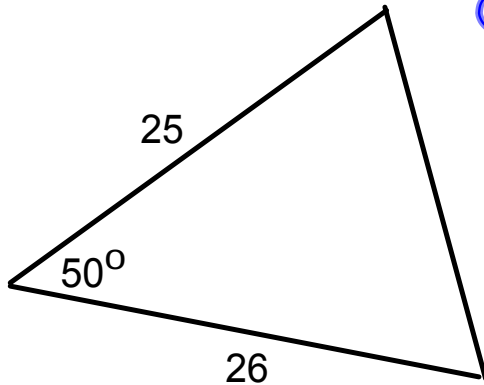
$$A = \sqrt{p(p-a)(p-b)(p-c)}$$

p : semi-perimeter

a, b, c : three sides of triangle

$$A = \sqrt{16.5(16.5-14)(16.5-12)(16.5-7)} \\ = \sqrt{1763.43} \approx 42 \text{ units}^2$$

Can Hero's Formula be applied to the triangle below? No.



Can you apply sine law to find 3rd side? No.

Another formula for the area of a triangle is the Trigonometric Formula:

$$A = \frac{ab \sin C}{2}$$

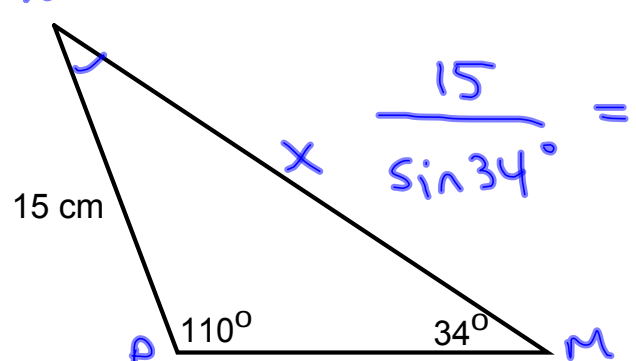
a, b : two sides of triangle

C : the angle between a and b

$$A = \frac{25(26) \sin 50^\circ}{2} \approx 249 \text{ units}^2$$

Find the area of the following triangles:

a) Use sine law first:



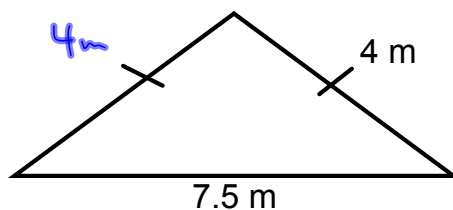
$\frac{15}{\sin 34^\circ} = \frac{x}{\sin 110^\circ}$

$$x = \frac{15 \sin 110^\circ}{\sin 34^\circ} \approx 25.2 \text{ cm}$$

$$\angle N = 180^\circ - (110^\circ + 34^\circ) = 36^\circ$$

$$A = \frac{15(25.2) \sin 36^\circ}{2} = 111.1 \text{ cm}^2$$

b)



$$P = 4 + 4 + 7.5 = 15.5 \text{ m}$$

$$p = \frac{15.5 \text{ m}}{2} = 7.75 \text{ m}$$

$$A = \sqrt{7.75(7.75 - 7.5)(7.75 - 4)(7.75 - 4)}$$
$$= \sqrt{27.24} = 5.2 \text{ m}^2$$