

# Periodic Functions

Goal:

- to **interpret** periodic functions
- to determine the **period** of a periodic function
- to use the **period** of a function to **make predictions** (extrapolate)

What is the most defining characteristic of a periodic function?

repeating pattern

(one cycle)

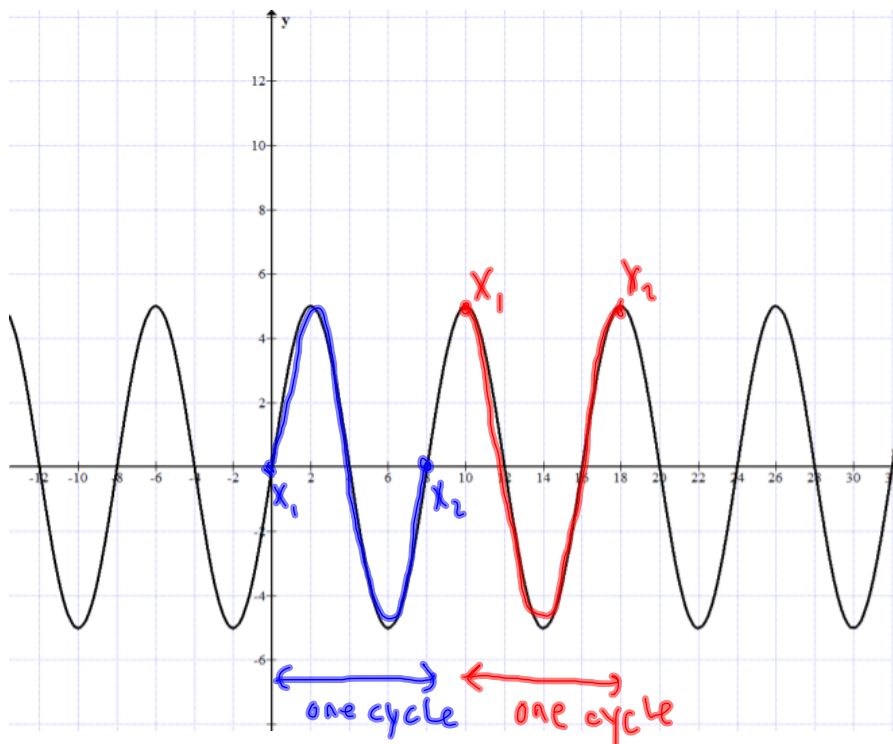
The length of the repeating pattern is called the function's **period**.

It is defined as:

$$P = x_2 - x_1$$

where  $x_1$  is the start of one cycle  
and  $x_2$  is the end of that cycle

What is the period of the function shown here:

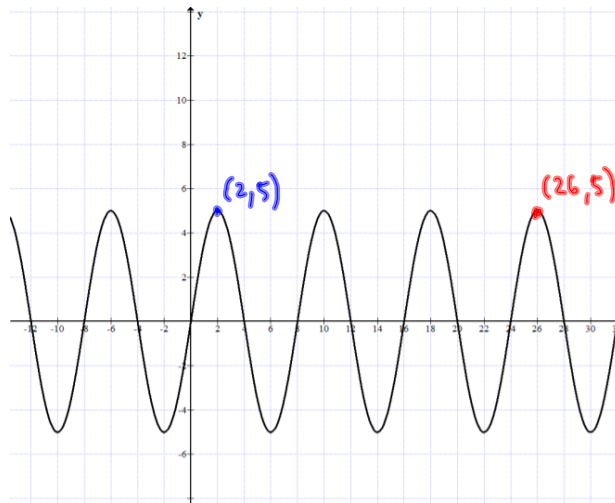


$$\begin{aligned} P &= x_2 - x_1 \\ &= 8 - 0 \\ &= 8 \end{aligned}$$

$$\begin{aligned} P &= x_2 - x_1 \\ &= 18 - 10 \\ &= 8 \end{aligned}$$

Once the period is known, we can use it to predict values that are not graphed.

For example, what is the y-coordinate when  $x=42$ , when  $x=112$  or at  $f(-24)$  ?



$$p = 8 \text{ units}$$

$$y = ? \text{ when } x = 42$$

take 42 and subtract multiples of 8 (the period)

$$42 - 8 = 34 \text{ when } x=26 \text{ } y=5$$

$$34 - 8 = 26 \text{ (from graph)}$$

and since  $x=26$  is two cycles away from  $x=42$  they have the same y-coordinate.

$$x=42 \text{ } y=5$$

more sophisticated:

$$\frac{42}{8 \text{ (period)}} = 5.25 \text{ cycles from } x=0 \text{ } \sim 5 \text{ cycles}$$

subtract 5 cycles from 42:

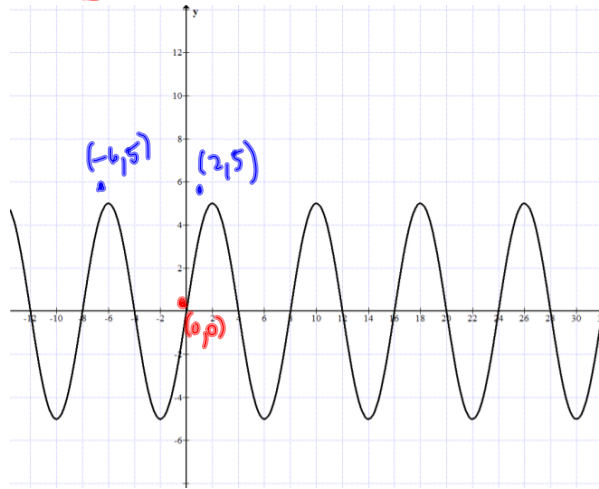
$$42 - 5(8) = 42 - 40 = 2$$

so  $x=2$  and  $x=42$  have same y-coordinate

$$(2, 5) \Rightarrow (42, 5)$$

Once the period is known, we can use it to predict values that are not graphed.

For example, what is the y-coordinate when  $x=42$ , when  $x=112$  or at  $f(-24)$ ?



$$x=112 \quad p=8$$

$$\text{so } \frac{112}{8} = 14 \text{ cycles from } x=0$$

$$112 - 14(8) = 112 - 112 = 0$$

so  $x=0$  and  $x=112$  have  
same y-coordinate

$f(-24)$  means  $y$  when  $x=-24$

$$\frac{-24}{8} = -3$$

so  $x=-24$  is 3 cycles left  
of  $x=0$

$x=0$  and  $x=-24$  have  
the same y-coordinate.

$$x=-24 \quad y=0$$

$y=?$  when  $x=-30$

$$\frac{-30}{8} = -3.75 \text{ cycles from } x=0$$

use 3 or 4 cycles

$$-30 + 3 \text{ cycles}(8)$$

$$= -30 + 24$$

$$= -6$$

$$\text{so } x = -6 \text{ and } x = -30$$

have the same  $y$ -coord.

looking at graph it is  $y = 5$

Using 4 cycles

$$-30 + 4 \text{ cycles}(8)$$

$$= -30 + 32$$

$$= 2$$

$$x = 2 \text{ and } x = -30$$

have same  $y$ -coord., looking  
at graph  $y = 5$

The formal math definition  
for periodic functions is :

$$f(x) = f(x+p) \quad \text{where } p: \text{period}$$

the  $y$ -coordinate based on  $x$  = the  $y$ -coordinate based on  $x$  plus on period

Homework : p. 55 #5

57 #9

Handout p. 25 #3