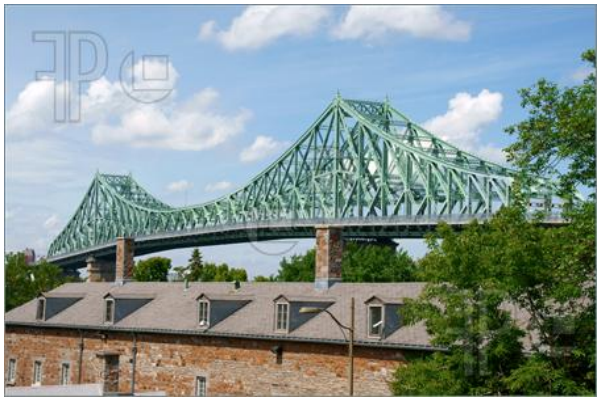
# Trigonometric Ratios

Goal:

- to learn what the trigonometric ratios are and what they represent

Champlain Bridge

Jacques Cartier Bridge



These are the two largest bridges in Montreal. The government is planning a 10-year $5 billion dollar replacement of the Champlain Bridge.

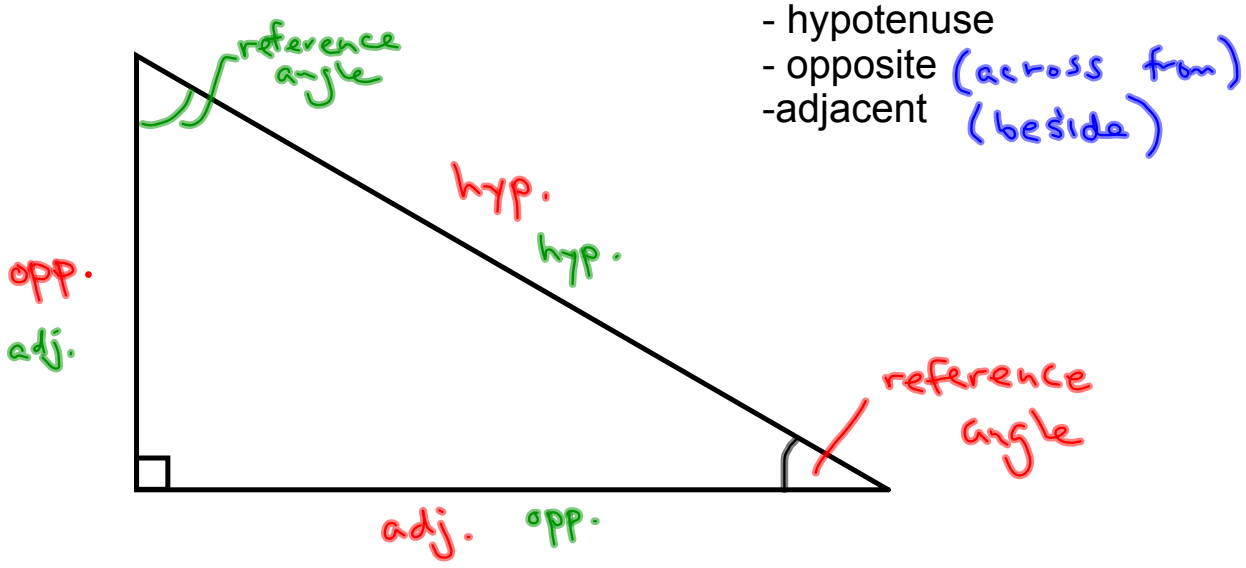
What shapes do you see when you look at the bridges?

Triangles!

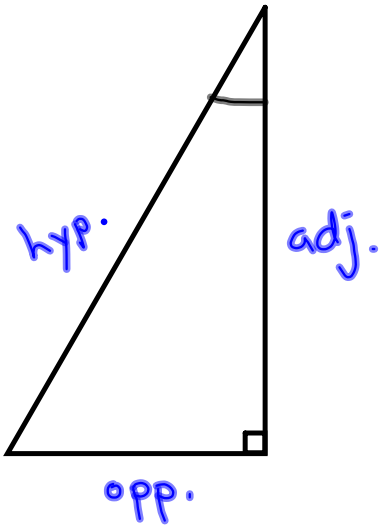
The importance of triangles in design has lead to a lot of study in mathematics. One of the most famous and well known is the concept of trigonometric ratios. The trig. ratios are used to relate the sides and angles in right triangles.

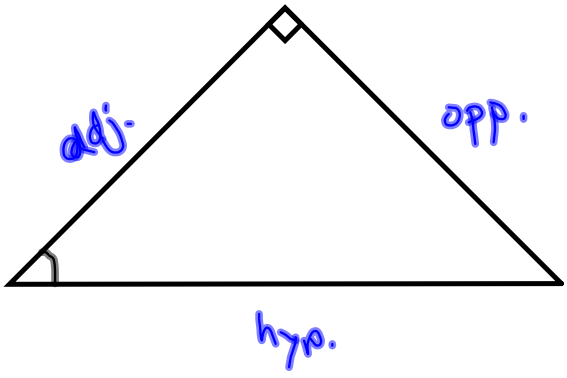
a ratio compares two quantities as a fraction.

We need to label each side:
- hypotenuse
- opposite (across from)
-adjacent (beside)



Label the hypotenuse, adjacent and opposite side for each triangle.



What is the relationship between the lengths of sides and an angle?

60°

| Triangle | Length of OPP | Length of ADJ | Length of HYP | $\frac{\text{opp}}{\text{hyp}}$ Fraction | $\frac{\text{opp}}{\text{hyp}}$ Decimal | $\frac{\text{adj}}{\text{hyp}}$ Fraction | $\frac{\text{adj}}{\text{hyp}}$ Decimal | $\frac{\text{opp}}{\text{adj}}$ Fraction | $\frac{\text{opp}}{\text{adj}}$ Decimal |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 19.6 | 11.5 | 22.6 | $\frac{19.6}{22.6}$ | 0.87 | $\frac{11.5}{22.6}$ | 0.49 | $\frac{19.6}{11.5}$ | 1.7 |
| 2 | 17 | 10 | 19.5 | $\frac{17}{19.5}$ | 0.87 | $\frac{10}{19.5}$ | 0.51 | $\frac{17}{10}$ | 1.7 |
| 3 | 13 | 7.5 | 15 | $\frac{13}{14}$ | 0.87 | $\frac{7.5}{15}$ | 0.5 | $\frac{13}{7.5}$ | 1.73 |
| 4 | 10.7 | 6 | 12.3 | $\frac{10.7}{12.3}$ | 0.87 | $\frac{6}{12.3}$ | 0.49 | $\frac{10.7}{6}$ | 1.78 |
| 5 | 9.5 | 5.5 | 11 | $\frac{9.5}{11}$ | 0.86 | $\frac{5.5}{11}$ | 0.5 | $\frac{9.5}{5.5}$ | 1.73 |
| 6 | 6.2 | 3.54 | 7.2 | $\frac{6.2}{7.2}$ | 0.86 | $\frac{3.54}{7.2}$ | 0.49 | $\frac{6.2}{3.54}$ | 1.75 |
| 7 | 3.2 | 2.1 | 3.8 | $\frac{2.2}{3.8}$ | 0.84 | $\frac{2.1}{2.8}$ | 0.55 | $\frac{3.2}{2.1}$ | 1.52 |
| 8 | 2.6 | 1.6 | 3.3 | $\frac{2.6}{3.3}$ | 0.78 | $\frac{1.6}{3.3}$ | 0.48 | $\frac{2.6}{1.6}$ | 1.625 |
| | | | | average | $\simeq 0.85$ | | $\simeq 0.5$ | | $\simeq 1.7$ |
| | | | | correct | 0.866... | | $\simeq 0.5$ | | $\simeq 1.7$ |

What conclusion can we make?

The ratios are the same for every right triangle with a 60° angle.

Will this be true for other right triangles?

Yes.

These ratios have special names:

the sine ratio compares opposite side to hypotenuse

$$\sin(\text{angle}) = \frac{\text{opp}}{\text{hyp}}$$

$\sin 60^\circ = 0.8660\ldots$

the opposite side is 0.866 times bigger (i.e. smaller) than hypotenuse.

the cosine ratio compares adjacent to hypotenuse

$$\cos(\text{angle}) = \frac{\text{adj}}{\text{hyp}}$$

$\cos 60^\circ = 0.5$

the adjacent side is half the hypotenuse

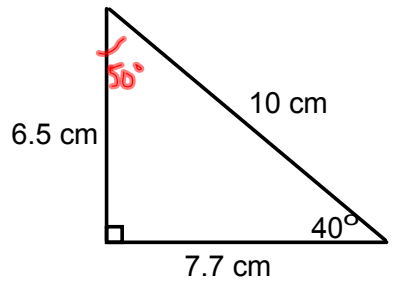
the tangent ratio compares opposite to adjacent

$$\tan(\text{angle}) = \frac{\text{opp}}{\text{adj.}}$$

$\tan 60^\circ = 1.7$

the opposite side is 1.7 times bigger than the adjacent side.

Determine the value of the three trig. ratios for the following triangle.




$\sin 40^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{6.5}{10}$

$\sin 40^\circ = 0.65$

$\cos 40^\circ = \frac{\text{adj.}}{\text{hyp}} = \frac{7.7}{10}$

$\cos 40^\circ = 0.77$

$\tan 40^\circ = \frac{\text{opp}}{\text{adj}} = \frac{6.5}{7.7}$

$\tan 40^\circ = 0.844$

calculator

$\sin 40^\circ = 0.6427\ldots$

$\cos 40^\circ = 0.7660\ldots$

$\tan 40^\circ = 0.8390\ldots$

$\sin 50^\circ = 0.7660\ldots$

$\cos 50^\circ = 0.6427\ldots$

$\tan 50^\circ = \frac{1}{0.839} = 1.1917\ldots$

What would the ratios be if the triangle was twice as big?

SAME!!

SOHCAHTOA

$\sin = \frac{opp}{hyp}$ $\cos = \frac{adj}{hyp}$ $\tan = \frac{opp}{adj}$

We will use the ratios (from our calculators) to find missing side measures.